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No. XXIII

OBSERVATIONS

ON

LANGUAGE.

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IT is a primary principle in all researches concerning language, *that nations will uniformly have such words, as express those ideas, which they wish to communicate.* Equally true is it, *that they will, for no length of time, retain any other.* Other words may be retained in books; but they will be lost out of the customary speech.

What is true, in this respect, of nations, is equally true of smaller classes of mankind. Thus lawyers, chemists, mariners, miners, &c. have a peculiar language of their own; made up of many words, unused, and ordinarily unknown, by other persons, belonging to the same nations. This partial, local language descends, also, from generation to generation in these classes of men, severally, as those of nations to their posterity.

This scheme admits of no exception; but is an absolute, as well as universal, law of language. From it many truths may be learned, which are of great importance in the philosophy of man; and may be learned with more ease, and more certainty, than in any other manner. For example; the character of a nation may with absolute certainty be extensively known from this source. The Latin language contains many terms, employed about *war;* that of the Greeks, many terms, expressive of the several ideas pertaining to *art*, and *science;* that of the Hebrews, many, denoting things, belonging to *morality* and *religion;* and that of the French,

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many, expressive of *the various modes, and circumstances, of polite intercourse.* Were we absolutely destitute of any history of these nations, and yet possessed of vocabularies, containing such words, as have been here alluded to; and were we assured, that the several classes of words had been in customary use among them, respectively; we should know with indubitable certainty, that the *Romans* were a military; the *Greeks*, a learned, and philosophical; the *Hebrews*, a religious; and the *French*, a polished people.

Equally evidential are the words, found in any language, of the pursuits, and other circumstances, of the people, by which it was used. After what has been said, little needs to be added on this part of the subject. If we know, that a man is a seaman, because he customarily uses nautical phraseology; we as certainly know, that a nation, whose language contains many commercial terms, is a commercial nation. The English language abounds in words, expressive of the rights, privileges, and duties, of freemen. This fact is a complete proof, independently of all history, that they have been much occupied in enjoying, and maintaining, these rights and privileges; and in performing these duties: in other words, that they have been a nation of freemen.

It follows, irresistibly, from this principle, that in all cases, where nations no longer wish to communicate certain ideas, which they have been accustomed to make, in some degree, subjects of their conversation, the words, which denote such ideas, will be dropped out of their speech. This is one of the great sources of what is called *the flux* of languages. The reason, why a multitude of words, formerly belonging to the *English* tongue, are not now lost, except so far as they are preserved in ancient books, is, that they have prolonged their existence in glossaries.

Nor is it less evident, that, whenever nations imbibe new ideas, which they wish to communicate, they will regularly originate such words, as are necessary to express them. A multitude of such words are continually introduced into the English language, at the present time. The flux of languages is extensively derived, al-

so, from *this source*. The present *chemical nomenclature* furnishes a strong example of a numerous, and systematical, addition, suddenly made in this manner to the principal languages of *Europe*.

From the observations, already made, we derive an obvious reason for the remarkable fact, *that one language contains words, to which there are no corresponding words in another*. A nation under a monarchical government has occasion to communicate few ideas concerning subjects, involved in a state of freedom. In a free State such words form a large vocabulary. When the people of *France* formed themselves into a republic, they were obliged to transfer many words, expressive of such ideas, out of the *English* into the *French* language. The modern nations of *Europe*, also, have for the same reason derived most of their technical terms from the *Greek* language.

In the *names of simple ideas*, all languages, it is believed, extensively agree; because most ideas of this sort are necessarily subjects of communication in every country.

Complex ideas are partly acquired from objects, presented to us in nature, or created in their existing forms by the power of GOD; or *they are formed in the mind*, in the three processes of *Composition*, *Abstraction*, and *Comparison*. The names of the former class will be extensively found in most languages; because the objects, whence they are derived, are found in most countries; and because they are, to a great extent, substantially alike, wherever they are found. Thus hills and mountains, valleys and plains, rivers, lakes, and the ocean, are the same things throughout the world. This is extensively true, also, of animals, vegetables, and minerals; and almost absolutely true of the appearances in the heavens. With respect to this class of objects, all languages must, in a great measure, necessarily agree.

The diversity in languages is chiefly confined to the names of Complex ideas, formed by the mind. By enlightened nations these invariably have been multiplied, exactly in proportion to their improvement in the progress of Society. By savages they are scarcely formed at all.

All ideas, of this class, are prompted by the exigencies of human life; by the demands of necessity, comfort, convenience, and pleasure. To a savage few demands of this nature occur. His comforts, conveniences, and pleasures, lie within a very narrow circle. By necessity, almost alone, is he, therefore, led to form such ideas: and the demands of necessity are always few.

Among civilized nations, (where necessaries are easily furnished.) comfort, convenience, and pleasure, multiply their calls without number. Hence the mind, ever intent upon answering them, is perpetually busied in forming those ideas, to which they conduct us. Hence men compound, abstract, and compare, endlessly: and the ideas, thus created, are increased ad libitum. In a general proportion, the words, which denote them, are increased, for the reason given above.

Savages, as truly as civilized men, have all the words, which express such ideas, as they have occasion to communicate: and a nation, or colony, falling back towards a state of savageness, although it will lose a great part of its former language, will still retain every such word.

From these observations it will be seen, that *the intelligence of any nation may be exactly estimated from its Vocabulary;* or from a dictionary, containing all its words. If the words, contained in such a book, are numerous; *and yet denote, severally, different ideas;* such a nation has, at some time, been extensively enlightened. If they are few; it is, and has been, proportionally unenlightened. The *English* language contains more such words than any other; and this fact proves with absolute certainty, that the nation, speaking it, possesses more ideas, which are communicated, than any other nation. The language of *Arabia* is said to be very copious. There are two ways, in which a language may become copious. One is that, already specified; in which words, denoting different ideas, are numerous. The other is the multiplication of synonimous terms. In the latter sense, the *Arabic* language may be copious. In the former sense, this is morally impossible; because the *Arabians* are, comparatively, an ignorant and uncivilized people. There was a period, when they were more

enlightened. At this period they multiplied words, denoting different ideas: and these may be found in the remaining books of that age. But the terms, now in customary use, are certainly, and necessarily, few. The multiplication of synonimous words is a proof, not of national improvement, but of addiction to pleasure and amusement.

The state of human languages has long been a subject of philosophical enquiry. "How," it is asked, "came the languages of mankind to be what they are: so numerous; so diverse in their words; and so unlike in their form, and construction?"

When we enter upon this inquiry, the first consideration of importance, that presents itself, is *the Confusion of languages at the tower of Babel.*

Who were the people, affected by this remarkable dispensation of Providence? To this question, I answer, *A part of the sons of Ham, joined together with others, who descended from Shem, and Japheth. Nimrod*, who led this enterprize, associated with him, first, his brethren and other near connexions; and then, all within his reach, who, with the true spirit of *modern Jacobinism*, were impatient of the government both of GOD and man. These people, having busily employed themselves, for a season, in hunting wild beasts under the conduct of *Nimrod*, began to turn their weapons against their fellow-men. For this purpose they found the continuance of their Union indispensable. They had become a numerous and formidable body, when GOD commanded *them*, together with the rest of mankind, to disperse in different colonies, for the purpose of peopling the world. This command they determined to disobey. Accordingly, they proposed to build a very lofty tower, and surround it with a city; and to make these the principal seat of their empire. In their company were probably numbered all the restless, violent, unprincipled, spirits of the age; and these must, to some degree at least, have been found among the descendants of *Japheth*, and *Shem*; for they have ever since been found in every Society, however peaceable, under heaven.

That all the descendants of *Ham* were not included,

we know; because we find *Abraham*, a descendant of *Shem*, a long time afterward understanding, and conversing easily, with the inhabitants of *Canaan*, and of *Egypt*, who were descendants of *Ham*.

Antecedently to the building of *Babel*, "*the whole earth was of one language, and of one speech.*" The Confusion of languages, in whatever manner, and in other respects to whatever degree, it was accomplished, extended so far, as we are directly informed, *that they did not understand one another's speech*, and were therefore obliged to desist from their favourite purpose of building the city and tower, which they had begun. Beyond this extent, the Scriptures give us no information concerning the subject.

If a person of intelligence should take a page of any work; and, as he read it, should insert two, or three, important words, or in many cases even one, in each sentence, from a language, which he did not understand; he would readily perceive, that the whole must be unintelligible to those, in whose language the composition was written. As GOD does nothing more than that, which is necessary for the accomplishment of his purposes; we should naturally conclude, that, if the dispersion of these builders had been the only object, intended by this dispensation, a change, amounting to little, or nothing, more than I have specified, was all, that was effectuated at this time. But, as this event was the beginning, and was intended to be the beginning, of the divisions of language, which have since taken place, the degree of this change cannot, without further evidence, be warrantably confined within the limits, here mentioned.

Should a project, said to have been originated in *Russia*, of *forming an universal nomenclature of human languages*, ever be completed; materials would be furnished for determining, exactly, their number and variety; and of course the degree, and the manner, in which changes in them have taken place. Until this shall have been done, the subject must remain in much obscurity.

The Second cause of change in language is *the involuntary tendency of man to vary his Pronunciation.* This is done from inadvertency, forgetfulness, and a general

indisposition to exactness, either of attention, or practice. From all these causes many persons pronounce many words of their own language imperfectly. Of this fact we are almost every day witnesses. Where any considerable body of people are insulated; or have little intercourse with the rest of a nation; an imperfect manner of pronouncing becomes ultimately diffused through that body; and terminates in a *distinct dialect of the language.* In this manner, it is believed, were originated the dialects among the *Greeks;* and in the same manner were formed the *Yorkshire* and *West-country* dialects in *England.* Similar instances exist in *France, Spain,* and *Germany.*

It has often happened, that those, who have originated such dialectic pronunciation, are, for a long period, almost absolutely separated from the parent stock. In every such case the dialectic pronunciation becomes unintelligible to the nation at large, in such a degree, as really to constitute *a new, spoken language;* although the words may still be *written* in the original manner. The *Koreans,* and *Japanese,* were colonists from *China;* and easily correspond with the *Chinese* in their written language; but cannot be understood by them, when they speak. The *Tonquinese,* and *Cochin-Chinese,* colonies also of the same nation, are said, to a great extent, to speak a different language; and this, it would seem, chiefly, or wholly, from a diversity of pronunciation: the *Chinese* language being still the substance of those, spoken in all these countries.

In the progress of this change of pronunciation it has often happened, that *nations, as well as tribes, have lost, finally, the sounds of several letters, and adopted those of several others.* Among these none are more remarkable than the Greek Θ, and X. The Romans could not pronounce the Θ, and therefore rendered Θεος Deus. This is equally true of the *French,* the *Dutch,* and many other nations. The *English* retain the pronunciation of this letter. X seems to be found among most, or all, nations, who distinguish their pronunciation by a peculiarly strong emphasis. Thus it is used by the *Irish, Scotch,* the *Germans,* and other descendants of the *Teutonic* tribes. It is also used by the *Iroquois,* and

some other *North-American* tribes. But it is lost out of the *English, French, Latin*, &c. In the *Irish* it is expressed by *gh;* in the *Scotch*, by *ch;* as *Lough, Loch.* The syllables, which terminate in *English*, in *gh*, were originally pronounced in the same manner. But the *gh* is now either *quiescent*, or converted into the sound of *f;* as *daughter. laughter.*

The *French u* is perhaps peculiar to that nation. If this be true; the words, in which it is found, when transferred to another language, will always assume a new pronunciation. The same may be said of their *nasal m*, and *n*.

A perfect alphabet, if we exclude the *French nasal m*, and *n*, contains 44 letters; if we include them, 46. These, distributed, in a natural order, are the following.

	Rough.		Middle.		Smooth.
Loud.	Be. Pe.	Low.	eth. eth.	Low.	L. el.
	De. Te.		ev. ef.		M. em.
	Ga. Ka.		ez. es.		N. en.
	Je. Che.		ej French. esh.		NG. eng.
					R. Smooth.
					R. Rough.
			Aspirates.		H. he.
					X. chi. Greek.

Vowels.

a in face,
in bat,
in barn.
e in fleece,
bet,
berm.
in fine,
fit,
ir.
o in bone,
home,
bott,
born.
oo in boot,
bush.
u in Bruce,
but,
burn.
u French.
u Indescribable.

Consonants.	
Rough	8
Middle	8
Smoot	6
Aspirates	
	24
Vowels	20
	44

Of these, all are found in the pronunciation of the *English* language, except the two last *u's*, and the Greek *X*, or the rough aspirate. The *French j* is sounded in *vision*, and other similar words.

The alphabet, said to have been brought by *Cadmus* into *Greece*, consisted of 16 letters.

From a comparison of these alphabets it will be seen, that, although that of *Cadmus* may, by making one letter the representative of more sounds than one, have been considerably enlarged in its actual effect, yet it must have been comparatively limited. The pronunciation of all languages, and dialects, is of course restricted to the number of letters, actually in use. These, compared with a perfect alphabet, are, perhaps always, defective in the number of their sounds; and most of them to a considerable degree. Wherever this is the case; a nation will be unable to pronounce those sounds in other languages, with respect to which its own language is defective. As many alphabets are seriously defective; this fact will produce, in the end, a great diversity of pronunciation. In one nation several sounds will be used, which are omitted by another; and several will be omitted, which are used by that other.

The *Commutation of letters* is another source of changes. The Germans use *d* where the English use *t;* and *b*, where the English use *p;* and *vice versa*. Thus they say *dumplers* where the English say *tumblers*. Commutations of this nature, as actually existing in the several nations of mankind, are considerably numerous; and have a considerable influence in varying pronunciation. The *Romans*, in transferring words from the Greek language into their own, changed

B into P, e. g. Βοσκω into *Pasco*,
F, Βρεμω into *Fremo*,
and V, Γιλβος into *Gilvus*.

They also inverted this order of mutation.

The *Greeks*, in making the same transferrence, changed the Roman

C into Γ: e. g. *Caius* into Γαιος.

The Romans themselves anciently wrote
C for *G:* e. g. *Acnom* for *Agnum*, and *Pucno* for *Pugno*.

In a similar manner they changed

D into *T*; and *vice versa.*

Anciently, *D* was written for *L*, as *Sedda* for *Sellă*, and *dingua* for *lingua*. It was also written for Θ, as *Ador* for Αθηρ.

The *Latin E* was changed into all the other vowels.

F was anciently written by the *Romans* for *H*, as *Fordeum* for *Hordeum*.

G was changed into *C*, and into *Z*; and

I into *U*, as *optumus* into *optimus*; and also into *E*, and *vice versa.*

L was sometimes changed into *N*, and sometimes into *D*.

The mutual commutations of *M* and *N* are too well known to be insisted on here.

It is also well known, that

N was sometimes changed into *L*, and sometimes into *R*.

O and *U* were frequently written for each other.

P was changed into *F* and *V*, Γ and *K*.

V was changed into *B* and *P*;

U into all the vowels; and

Ionic.

Z into *G*, as *ολιζον* for *ολιγον*.

Doric.

ζευω for γευω.

From these changes, which might be pursued to a greater extent, and of which many more examples might be given, it is evident, that language may, and in many instances must, have been so greatly altered, from this source alone, as to become unintelligible to those, by whom the changes had not been made.

This, however, was not the whole amount of these alterations. *Words were often written*, and by the *Greeks* generally, *in such a manner, as to make them accordant with the genius of the language, into which they were transferred.* Thus *Ormuzd*, *Mithr*, and *Ahriman*, the names of the *Persian Triad*, were written by the Greeks *Oromasdes*, *Mithras*, and *Arimanios*; *Khosrau*, or *Caikhosrau*, Κυρος, and by us *Cyrus*; *Cambakhsh*, *Cambyses*; and *Shiruyi*, or perhaps *Shirshah*, *Xerxes*. Let

this change of mere writing be carried through a language; and it will become a new one at once.

It would be a curious, and not uninstructive, employment, *to trace the progress of words through the various deflections, produced by changes in the mode of writing merely;* and that in cases, where the original is all along perfectly known. I have no sufficient means of extending this kind of research far. The few following instances, may not, however, be without their use.

Examples of the different modes, in which the following words in the Hebrew language were written in kindred dialects.

Dialects.	*Father.*	*Heaven.*	*Earth.*	*Bread.*
Hebrew,	Ab.	Schamaim.	Arez.	Lechem.
Rabinical,	Av.	Idem.	Aretz.	Idem.
Chaldaic,	Abba.	Schmaja.	Ara.	Lahkma.
Samaritan,	Ab.	Schamaim.	Aretz.	Lechem.
Syriac,	Aboh.	Schmajo.	Arho.	Lachmo.
Arabic,	Aba.	Ssemavati.	Ardhi.	
Vulgar Arabic,	Abu.	Ssamvat.	Ardh.	
Malay, Bengalee,	Bappa.			
Ethiopic,	Abi.	Samai.	Myrdrni.	
Amharic,	Aba.	Ssamai.	Myrdrm.	
Shilhic,	Baba.			
Rhætic,	Bap, Pap.			
Sardic.	Babbu.			
Charaibic,	Baba.			

Examples of such words, derived from the Greek and Latin.

	Father.	*Heaven.*	*Earth.*	*Bread.*
Greek,	Πατηρ.			
Latin,	Pater.	Cœlum.	Terra.	Panis.
French,	Pere.	Ciel.	Terre.	Pain.
Italian,	Padre,	Cielo.	Terra.	Pane.
Sardian,	Pare.	Quelo.	Id.	Pa.
Spanish,	Padre,	Cielo.	Tierra.	Pan.
Catalonian,	Pare.	Cel.	Terra.	Pa.
Portuguese,	Pae, Pay.	Ceo.	Terra.	Pao.
Scotch,	Fäther.	British,	Dhayar.	
English,	Father.	Welsh,	Izer.	Pæne.
Runic,	Fader.			

Dialects.	*Father.*
Anglo-Saxon,	Fæder, Fader.
German,	Fater, Fatter. Bater, Batter.
Belgian,	Vader.
Dutch,	Vader.
Orkney,	Favor.

From the Teutonic.

	Heaven.	*Earth.*	*Bread.*
Gothic,	Himina.	Airthai.	Hlaif.
Runic,	Himium.	Jordaune.	Brodh.
Anglo-Saxon,	Hofenum, Heofnas, Heofunum.	Eorthan, Eortho, Eorthe, Heortho.	Hlaf.
Franc,	Himili.	Erdu.	Zuthi.
Allemanic,	Himile.	Erdo.	Brot.
German,	Hymel, Hemmal, Hemmele.	Erd, Earda.	Brot, Brodt, Braud, Brod.
Belgic,	Emelrike, Hemel.	Erdrike, Aerd.	Broet, Broot.
Frisian,	Hymil. Hiemmel.	Urtricke, Jorde.	Brae. Breea.
Icelandic,	Himne.	Jorde.	Braud.
Danish,	Himmel.	Jorde.	Brod.
Swedish and Norwegian,	Himmel.	Jorden.	Brod, Brouta.
English,	Heaven.	Earth.	Bread.

From the Sclavonian.

	Father.	*Heaven.*	*Earth.*	*Bread.*
Sclavonian,	Ottse.	Nebu.	Zemlyi.	Kruka.
Russian,	Otshe.	Nebesi.	Zemli.	Kleb.
Bohemian,	Ottse.	Nebi.	Zemi.	Hleb.
Dalmatian,	Otse.	Nebu.	Zemlyi.	Kruh.
Croatian,	Otse.	Nibeszih.	Zemli.	Hlib.
Vandalian,	Wotz, Woschzi.	Nebui, Nebu.	Semi, Zemi.	Chlieb, Kleb.
Bulgarian,	Otskye.	Nebu.	Zemli.	Kruh.
Nova-Zemblian,	Otcse.	Nebezi.	Zemli.	Klyb.
Servian,	Otse.	Nabeszih.	Zemli.	Hlib.
Carniolic,	Otze.	Nebeszih.	Zemlyi.	Krah.
Lusatian,	Voshe.	Nebie,	Zemie,	Klib.
Livonian,	Tabes.	Debbes.	Summes.	Mayse.
Polish,	Oycze.	Niebe.	Ziemi.	Chleb.

Names.

	James.		*John.*
Hebrew,	Yakob.	Hebrew,	Johanan.
Greek,	Ιακωβος.	Greek,	Ιωαννης.
Latin,	Jacobus.	Latin,	Johannes.
French,	Jacques.	French,	Jean.
English,	James.	English,	John.
Italian,	Giacomo.	Dutch, vulgar,	Hans.
Spanish,	Iago.	Italian,	Giovanni.
Abyssinian,	Yagoube.	Spanish,	Juan.

A fourth cause of changes in Language, is found in *the State of Society.*

It must, I think, be admitted, that, if a nation, or perhaps as a better example, a tribe, were to continue for many ages in the same spot, where the same natural objects were presented to every succeeding generation; in the same state of manners, laws, and religion; and, universally, were to be possessed of the same ideas, and the same occasions to communicate them; *they would keep their language unaltered.* But, with every change in society, an alteration in their language will take place. This part of the subject has been, to some extent, illustrated in laying down the scheme, at the commencement of these observations. I will attempt further to illustrate it in the following manner.

Suppose a ship, from a country in a state of complete civilization, containing a numerous body of passengers; men, women, and children; to be wrecked on an uninhabited shore; and these people enabled to find sufficient subsistence, and to inhabit the country, through an indefinite series of generations. What effects would these events produce on their language? A satisfactory answer to these enquiries will be found in the following observations.

1*st. They would in a short time lose a great part of their language.*

It has been observed above, that men have those words in their customary speech, which express such ideas, as they have occasion to communicate; and no other. These people, being without books, and metals,

would soon lose their knowledge, arts, and civilization; and with them the great mass of their ideas. In other words, they would become, in a moderate number of years, absolute savages. With these ideas, they would of course lose the words, by which they were denoted. Particularly, they would lose the names of all the parts, and of all the furniture, of a house, except such as were retained in a *weekwam;* of all the utensils, and of all the parts of dress, except such as are used by *Indians;* and of all employments, beside such, as *Indians* pursue. Out of their language would vanish the terms, which belong to agricultural, mechanical, and manufacturing, business; to navigation, and commerce; to the science, and business, of government; and to the liberal arts, and sciences; the greater part of those words, which pertain to religion, and morals; and to decency, and refinement of manners. Universally, they would lose almost all those words, which denote ideas, formed by composition, abstraction, and comparison. But these various sorts of words constitute almost the whole language of a civilized people.

They would also lose the names of those natural objects, presented to them in the country, whence the colony emigrated, and not found in their present residence.

2*dly. At the same time, they would form new words, to express such natural objects, as were different from those, which they had known before, and such employments, as were new to them; and, universally, for such new ideas, as they had occasion to communicate.* These, however, would be few.

Thus their language would by this cause become greatly changed. Most of it would vanish: a part would be retained; and a part would be new.

Among the words, which they retained, would in all probability be found *the names of near, and important, natural relations;* as father, mother, brother, sister, &c.; *the personal pronouns; the names of the most common natural objects, and of the most familiar, and important, actions.* The *Mohekaneews* have the same personal pronouns with those of the Hebrew language; and in all

probability derived them, at some period or other in the history of man, from that language: such a fact being hardly attributable to chance.

What would be true concerning persons, thus shipwrecked, must be equally true concerning a body of people, separated from a civilized stock, and wandering into a wilderness so far, as to lose their intercourse with that stock. All the effects produced on the state of society in the case, which I have supposed, would be effected here. Such a colony would lose, ordinarily, the the knowledge of the metals, and of course the employments, and ideas, to which they gave birth. Of civilized men they would speedily become savages; and would sink their pastoral, agricultural, and commercial employments, in hunting, fishing, and war. Of course, their language would change in the very manner, which is here supposed.

The colonies, which emigrated from the family of *Noah*, were some of them very differently situated from others, with respect to the principal subject of this discourse. Some of these carried with them all their arts and civilization; and of course their language; that is, chiefly. Others speedily became mere savages, and hunters; and were the first among mankind, who sustained this character: there being no reason to suspect, that such a state of society, as is usually denominated the hunting, or savage, state, existed among the antediluvians. The language of these colonies underwent the very changes, which have been mentioned above, in the case of the supposed *shipwreck*. Other colonies, still, in various degrees lost more of their civilization than the former, and less than the latter, of the classes, already specified. These retained their language in different degrees. As these several colonies settled in very different countries, and climates, where different sets of objects were presented to their view; as in these countries they pursued very different employments, and had occasion to communicate very different ideas; the parts of their languages, which were new, must have been very different. In some instances they must have been almost wholly unlike; in others very similar.

Should we now suppose the descendants of this shipwrecked company, or of one of these colonies, which had fallen into a state of barbarism, to emerge again into a state of civilization, and science: Their language, thus changed, and thus barren, would in an inverse order undergo another change, equally great; as their ideas, particularly those derived from composition, abstraction, and comparison, would be greatly multiplied; those, I mean, which they would wish to communicate: and for all these, words would be devised. Should they derive their improvements from the books of another nation; these words would extensively be taken from the language of that nation: and this seems to have been the only course of mankind, in their progress out of the savage and civilized states. In this view of the subject it is perfectly evident, that, *in consequence of these two great revolutions, their language would become in a great measure new.*

A fifth cause of changes in language is found in *Intercourse with other nations.* Persons, who have much intercourse with each other, always become acquainted in some measure with each other's language. Of these, a few, comparatively, will learn the respective languages well, and will keep them distinct. The many will acquire them imperfectly; pronounce them badly; and mingle them together. The words of one language, however, will in all such cases be transferred to the other.

The most remarkable example of a mixed language, springing from such intercourse, of which I have heard, is *the lingua Franca.* This, as I understand, is a medley of various languages, spoken by no nation in the world; understood by no nation; and accordant with no grammatical analogy, or legitimate pronunciation. Yet it is spoken, and understood, on the borders of the *Mediterranean*, by such inhabitants of different countries as live in trading towns on the shore, and have intercourse with its several nations. A more striking proof cannot be wished, that mankind will always conform their speech to the exigencies of life.

It ought to be observed, that, when words are from

this cause transferred from one language to another, they are generally made to follow the analogy of the language, in which they are interwoven; yet not so absolutely, as to prevent them, if considerably numerous, from increasing its anomalies.

The sixth and last cause of such changes, which I shall mention, *is Conquest.*

Mankind have been almost ever at war; and conquests have been made in every spot on the globe. Wherever the conquerors have continued for a length of time, and established their domininion, they have introduced a material change into the language of the conquered. The *Romans* spread their language over their whole empire; and made it the written language of all their western, northern, and of nearly all their southern, provinces. The *Saxons* in a great measure exterminated the language, spoken in *England* before their arrival; and made their own tongue the principal source of the present *English*. *Canute* introduced a number of *Danish* words. *William*, the *Norman*, sprinkled the language with *French* words: and thus have other conquerors affected the languages of the countries, which they subdued.

In conquered countries, the laws, and legal processes, being uttered, and directed, by the conquerors, will usually be in their language. The knowledge, and use, of this language, must in a considerable degree, therefore, be indispensable to the comfort, and even to the safety, of the conquered. Hence it will be extensively learned by them. Multitudes will also acquire it from fashion; from a desire to resemble their rulers; from the hope of being employed in departments of the government; from the prospect of obtaining lucrative business; and from various other motives. When conquerors have ruled, for a great length of time; the changes, wrought in language by their influence, become permanent; and the language will never return to its original state. The *Shanscrit* is now a learned language in *Hindoostan*; and even the Bengalee, which seems to have followed it in the country of *Bengal*, sustains this character in a considerable degree. In the *Lowlands* of *Scotland*, the

English, introduced by conquest, has chiefly supplanted the *Gaëlic;* and has retained its place, in a dialectic form, down to the present time.

There is one subject, of considerable importance in this investigation, which has scarcely been touched in the course of these remarks. This is *the diversity of the manner, in which languages are constructed.* The things, in which, so far as my knowledge extends, this diversity has been most conspicuous, are *the variations of meaning by prefixes, and suffixes;* and *the expression of relations,* on the one hand, *by terminations of nouns,* and, on the other, by *prepositions;* and *the conveyance of diversities of Action,* in some languages by *the various endings of verbs,* and in others by *auxiliaries.* Of the first of these it will be unnecessary, here, to say any more, than that it existed, not only in the Hebrew, but in the language of the *Mohekaneews,* that of the *Araucanians,* and, if I mistake not, a number of others. With respect to the diversity in the last cases mentioned, it is well known, that the *Greek* and *Roman* languages depended, to a great extent, for the expression of relations, on the terminations of their nouns, and of the shades of action on those of their verbs; and that the modern languages of *Europe* denote, chiefly, the former by prepositions, and the latter by auxiliaries.

Concerning this subject, I observe in the first place, that *the use of auxiliaries seems chiefly to have been a modern invention.* The *Greeks* used the substantive verb, in a very small degree, as an auxiliary. The *Romans* adopted this form of phraseology more extensively: but neither of these nations had any other auxiliary; and in the use of this, both were comparatively confined. In the *English* language *there are no less than eight auxiliary verbs; and these are combined in a prodigious variety of forms.* But *neither of the languages,* from which the *English* is derived, was constructed in this manner. The *Roman,* and to a considerable degree the *Saxon,* depended on terminations to express the relations, and variations, of thought. We have not, therefore, followed the analogy of either of the parent languages; but have created a new one, and that, inde-

pendently of the anomalies, with which our language has been disturbed by the confluence, and mixture, of many streams. Most of the modern languages of *Europe* have followed the same course. The effect, therefore, seems to have flowed from a cause, common to all the nations, by which these are spoken.

Secondly. *It is admitted on all hands, that the second of these modes is the most favourable to Perspicuity; and the first, to Impression.* The *Romans*, and *Greeks*, were unquestionably far more solicitous than we are to make deep impressions on those, to whom they wrote, or spoke; and far less, to communicate clear and distinct ideas. For the former of these purposes *their* languages were more fitted, and for the latter much less, than *ours*. The *English* Verb, particularly, is by its auxiliaries moulded into a wonderful variety of forms: all of them adopted, to express real varieties of existence, and action; and all of them actually expressing these varieties. The *English Active Verb*, in the first person, and indicative mode, is varied through *four forms:* all of them expressing different shades of thought. The *Latin Verb*, in the same circumstances, has only *five forms;* and the *Greek*, *eight: three of them* superfluous, and useless, or very nearly so. The *English Potential Mode* is probably a peculiarity, and the greatest ornament, of our language; expressing shades of thought, of vast importance to minds of superior intelligence, and incapable of being expressed, even in circumlocutory phraseology, by either the Latin, or the Greek. Much less were they expressible by the variations of the Verb in these languages. It is hardly necessary to observe that neither of these languages had a *potential mode*. The *Greeks* had an *Optative;* but it was of very little significance, and very little use. The potential mode in the English is indispensable to exact and superior ratiocination. The forms of the verb in this mode amount to no less than 32.

The forms in the *Subjunctive*, every one of which has its use, are equal in number to those in both the preceding modes: viz. 46.

This example shews, in the strongest possible manner, the unrivalled attention, paid by the *English* nation

to exactness of thought, and to perspicuity, and precision, of language.

The number of *Prepositions* in our language is much greater than that, in both the *Latin* and *Greek* tongues. I do not know this number exactly; but it is not far from *fifty*. As the whole use of prepositions is to express with exactness the relations between objects; this multiplication of them is another remarkable proof of the same attention, on the part of those, by whom it is spoken.

Both the *Greeks* and *Romans* were sensible of the imperfection, with which relations were expressed by the terminations of their *Nouns;* and therefore adopted a considerable number of prepositions, to aid themselves in expressing these relations more perfectly. But this imperfection is much more strongly visible in a single example. By *the ablative case* in the Latin language were customarily denoted the several relations, which we express by the prepositions *at, by, from, in, on, with.* What a degree of ambiguity is involved in this single fact: especially when we remember, that it is applicable to every noun: and how perfectly is this ambiguity precluded by the use of these prepositions? I know there are men, who, either from taste or the affectation of it, or from the preference of impression to truth, would willingly decline the advantages, accruing from this and other similar sources to the intellect; and rest satisfied with the superiour neatness of a language, resting its meaning on terminations. With these men I shall not enter into a controversy; but shall content myself with merely saying, that, in my own view, reason is more valuable than passion and imagination; clearness of thought, than impression; and perspicuity and precision of style, than beauty.

The vast multiplication of *Conjunctions* in our language, for the purpose of expressing with exactness the connections, and dependencies, of thought, is another proof, that the same object was strongly in view; and a part of the same design. These, if I mistake not, are more numerous in our language than in both of the others.

For these reasons it seems probable, that *this change*

of Construction was derived from design; not indeed a design, systematically examined, and adopted, but forced upon the mind by its circumstances; peculiarly by the necessity, which was generally felt, of communicating truth, and of devising the modes, by which it might be exactly distinguished.

Having now, in some measure, shewn how languages are varied, both as to their terms, and their construction; it only remains to enquire *how many languages, radically distinct, there are in the world.* This enquiry is not intended to elicit any other than a loose, indefinite answer; which is plainly the only one, capable of being given. Yet this, perhaps, may be so given, as to induce a belief, that the whole number is incomparably less than has been generally thought.

In *Europe* there seem originally to have been but three distinct languages: the *Celtic*, the *Teutonic*, and the *Sclavonian.* The *Greek* and *Latin* languages appear, evidently, to be mixed, and not original. The *Greek* contains many *Celtic* words: and the original inhabitants of Greece were, I think, very evidently of the Celtic family. Their language was first mixed by the arrival of strangers from *Egypt*, and *Phœnicia:* chiefly, as I believe, *Cushite shepherds.* The *Latin* was partly *Celtic;* this being the language, spoken by the first inhabitants of *Italy;* and partly a corrupted *Greek*, derived from the colonies of that nation, which settled in *Magna Græcia.*

In *Asia* there were originally the *Hebrew*, and its dialects; the *Arabic*, the *Syriac*, and the *Chaldee;* the original *Persian;* the same, according to the evidence, exhibited by Sir William Jones, with the *Shanscrit;* the *Malayan;* the *Tatar;* at first probably the same throughout all the countries in the north of Asia; and perhaps the *Chinese;* not improbably derived from the *Tatar*, and mixed. The *Malayan* seems to be the radical language of most of the Islands. The original languages of *Egypt* and *Abyssinia* were, I think, evidently dialects of the *Hebrew;* or, perhaps more properly, the *Hebrew itself*, and *all its kindred tongues*, were *dialects of the one Original tongue*, spoken by *Noah, and his family.*

The languages of *Barbary* have in them nothing original. Those, spoken in the African countries, south of Barbary, are so little known, that nothing can be asserted concerning them. From analogy, I suspect, it will be found, that they are not very numerous.

In *North-America*, that of the *Iroquois*, that of the *Mohekaneews*, that of the *Choctaws*, and the three or four languages of *Mexico*, seem to be pretty well distinguished. In *South-America*, the *Peruvian*, the *Araucanian*, some languages in *Brazil*, that of the *Charaibes*, and perhaps one or more in *Terra Firma*, and one or more in *Patagonia*, are probably not far from the whole number. So far as this representation may be admitted as just, it will be seen, that the number of languages is not very great. There are, undoubtedly, more than these: some, spoken in *New-Holland*; some, in *Amazonia*, &c. Yet, after all allowances, the number, it is believed, will be comparatively small.

Travellers, and other persons, have, it seems to me, been accustomed to multiply the languages of mankind upon slender evidence. It is not long since, that many languages were supposed to be spoken in *New England*. It is now ascertained, that there was but one: that of the *Mohekaneews*. This, as it appears, was the language of all the tribes, the *Iroquois* excepted, between the *Atlantic* and the *Missisippi*, the *Potomac* and the *St. Lawrence*. It is also proved to be spoken by various tribes beyond the *Missisippi*; and by one within 400 miles of the *Pacific Ocean*.

I shall now leave it to the judgment of the reader to determine whether the causes, which have been here assigned, will not, to a considerable extent, account for the variety of languages, both in terms, and construction, so far as their state has been hitherto known.

No. XXIV.

ON

LIGHT.

To the Secretary of the
Connecticut Academy of Arts and Sciences.

SIR,

IT is known to most of the gentlemen, who usually assemble at our meetings, that my eyes have long been weak. In some respects, the disease, with which they are affected, is perhaps peculiar. At least I have never seen some of the effects, which it produces, described in any publication. As they seem to me to elucidate, in a degree, the nature of light; I beg leave, through your good offices, to communicate this paper to the Academy.

The disease, whatever else may be its nature, appears to consist, partly, in an uncommon irritability of the optic nerve, and a consequent, uncommon susceptibility of impressions from light; and partly, of a relaxation, and enlargement, of the lymphatic vessels. Whether my views of the disease are just, or not, I am not desirous to determine: with regard to its effects, I cannot mistake. Among them this is one; that at all times, when it is dark, whether my eyes be closed or not, *light, distributed into innumerable little stars, or sparks, is seen floating over the field of vision in a great variety of directions.* In the appearance of these luminous points *there is little regularity.* They become visible, and disappear, in many instances, almost in the same spot. In many others, they move over a considerable distance, before they vanish. The most general, and that, which approaches nearest to a regular, motion, is from the upper, towards the lower, limit of vision. These stars

appear in succession, whenever I am surrounded by darkness, whether in a dark room in the day time, or in the darkness of the night. Their existence I attribute to the *vascular circulation.* This, if I conjecture right, occasions in my eyes *an unnatural, and excessive, pressure upon the optic nerve.*

Whenever my eyes feel crowded, (a sensation, perfectly distinguishable, particularly from the frequent repetition of it at intervals, in some degree regular;) around the centre of vision *flashes of light, distantly resembling what is called a glory, but with little regularity of form,* take the place of these luminous points; and succeed each other with a considerable degree of rapidity. When I am in bed, and lie upon my back; they become intense, and vivid. If I turn upon my side, the brilliancy begins immediately to lessen, and after some time fades chiefly away. These appearances I attribute to an increased pressure, in both cases, upon the optic nerve. This pressure I suppose to be greatest, when I lie upon my back, and to produce the peculiar splendour of the flashes, at that time. When I lie upon my side, and my eyes are in a horizontal position; I suppose the pressure to be less, and the splendour for this reason to be diminished.

In many instances also, when my eyes are less crowded, *luminous clouds;* (phraseology, which better accords with the appearances, than any other, that I can think of;) commencing near the exteriour limit of vision, and irregular in their form, but filling up almost the whole field, contract themselves with considerable rapidity toward the centre, and there vanish. Their colour, towards the exteriour limit, is very frequently a reddish brown. As they advance inward, it becomes a strong *Saxon blue;* and in the end *a green,* which is often vivid. Of these clouds, floating inward, lessening, and vanishing, there is in such cases, an uninterrupted succession. Their luminous appearance I attribute to an unnatural degree of pressure. Of their figure, and motion, I am unable to assign the causes.

Whenever it is dark; if I turn my eyes suddenly to the right, and left; on the side of each, opposite to that, towards which I turn them, there is formed *a sudden,*

and brilliant, circlet of light; somewhat irregular, and a little yellower than that of the sun. This fact I attribute to the *increased* pressure on this part of both eyes.

All persons, who look at the sun, will perceive a bright image of this luminary, remaining in their eyes after they are turned away; more vivid, when the eyelids are shut; less, and less, when they are open. Persons, who have weak eyes, will derive this image from objects, which are in a moderate degree luminous; and will perceive it much longer, than those, whose eyes are not impaired. On my own such an image is impressed by every thing, which has any lustre; and, when they are most diseased, by every thing, which is of a light colour. When the image is vivid; it continues several minutes; but becomes gradually fainter, until it disappears. If, during the progress of its decay, I close my eyelids so, as to press them somewhat hardly; the image always becomes somewhat brighter, than it was immediately before. When this pressure is removed; it immediately becomes fainter.

From the uneasiness, which this disease very often occasions, I have not unfrequently been induced to press my eyes with my fingers. When the pressure reaches to a certain degree; *the whole field of vision becomes instantaneously luminous and bright; resembling in appearance a circular, polished plate of silver; covered with small drops of water, glittering with its lustre.* If these drops should be supposed suddenly to rise from behind the plate, and to move on its surface with rapidity, and without much regularity, the resemblance will, I think, be nearly complete. Sometimes I have increased the pressure beyond this degree. In this case, the drops of light have immediately disappeared; and *the field of vision has assumed exactly the aspect of the sun;* circular in its figure; uniform in its appearance; equally bright; and the light exactly of the same hue.

In some of these instances I have brought my finger suddenly, and carelessly, against the ball of the eye. In consequence of the pressure, produced in these cases, *a flood of light has instantaneously overspread the field of vision; exhibiting all the colours of the rainbow, not*

less distinct, nor less vivid, than those of the prismatic image.

At times, the disease, which has been so often referred to, is attended with peculiar affections, arising from an unnatural accumulation of lymph, or some other fluid, either above, or on the left side, of my left eye. Among these affections are the following.

At times, *a bright spot, irregular in its figure, and like the sun in its lustre, and hue,* appears in the field of vision; brilliant even in the day time for a considerable period, in each instance; *uniform* in its aspect; and lasting from a quarter of an hour to two hours. In many instances these spots have, during a part of the process, lost their lustre, and suddenly become black. The lustre of the spots I attribute to an unnatural pressure on a part of the optic nerve; the blackness, to an increase of that pressure, so great, as to interrupt for a season the sensibility of the nerve in that place.

When the disease in this form has reached the utmost height, which I have experienced; *the field of vision has been crowded with lucid appearances, sometimes fixed, and sometimes moving.* They have also been of different forms. Those, which were fixed, have resembled a confused collection of pieces of white glass, or rock crystal, cut in the form of parallelopipedons, bevilled on both sides, and at both ends; and then held up to a sky light. In several instances I have almost lost my sight, when this affection has become intense; and supposed myself on the verge of an apoplectic fit. But, in every case, vigorous exercise has in a short time dispersed these unpleasant symptoms. This effect I attribute to the mere increase of the circulation.

The facts, already mentioned, may be sufficient to warrant the following conclusions.

1st. *All the appearances, denoted by the word,* VISIBLE, *are inherent in the nature of the Optic nerve; and not in the nature of light; nor in the nature of the objects, by which* IT IS TRANSMITTED TO THE EYE. We are prone, if I mistake not, to suppose, that there is a lustre in the element of light; that it is coloured with seven distinct hues; and that it diffuses over the face of nature

the fine tints, which in an infinite variety adorn the universe around us. This, at least, has been the current of my own apprehensions. But from the facts, which have been recited, it is certain, that the splendour, and the colours, which we attribute to light, are the result of mere pressure. But there are neither colours, nor splendour, in pressure; nor in the finger, from which it proceeds. These extraordinary, and delightful, phenomena have their seat in the optic nerve; and exist in all their diversities, only as effects of the variations of pressure. Light itself has no more brilliancy than the finger. Its whole power is merely that of *gentle, and imperceptible, pressure*, or *impact, upon the optic nerve:* and this pressure, like that of the disease, and like that of the finger, awakens in the nerve the sensation of luminousness.

2dly. *Light is matter, and not a Quality of matter.* No degree, or kind, of impulse, whether such as we call pressure, or any other, can possibly be the effect of any *mere quality.* Both weight, and motion, are indispensable to its existence. Motion and weight, therefore, are certainly attributes of light.

3dly. *The Colours of light are the result of nothing, but different degrees of pressure, and impulse.* The colours, mentioned as seen in the case, specified above, were exactly the same with those of the prismatic image; equally bright; and, of course, sensibly brighter than those of the rainbow. In this case they were certainly derived from different degrees of pressure. There is not the least reason for attributing the effect to any thing else, when these colours are seen in the prism, or the rainbow. As the three strongest colours are less easy, and, when viewed for some time, more painful, than the other four; it is probable, that they are produced by a stronger impulse, or greater pressure.

4thly. *This difference of impulse can be derived from nothing, but the weight, or in other words the mass of matter, and the velocity, or both, of the particles of light.* As they come to us from the sun, perfectly, or almost perfectly, blended; (there being, for some reason or other, a small predominance of the yellow;) it seems probable, that their velocity is the same. Otherwise, it will

be difficult to explain why every pencil of rays is not in some degree particoloured; at least momentarily. I should, therefore, incline to the opinion, that the rays, which produce the different impressions of colour, differ in their weight.

If I mistake not, the facts, which I have mentioned above, all lend their influence, with some diversity indeed, to the support of these positions.

I am, Sir, yours, &c.

T. DWIGHT.

No. XXV.

DEMONSTRATIONS
OF
STEWART's PROPERTIES
OF THE
CIRCLE.

By THEODORE STRONG,
Professor of Mathematics and Natural Philosophy in Hamilton College.

THE following propositions are to be found in Dr. Rees' Cyclopædia, under the article "*Circle.*" They were proposed to me for solution. Having examined and found them to be very curious, and connected by one general principle, I have, with a view of contributing my mite to the advancement of science, thought proper to communicate the following demonstrations of them to the Academy. I have succeeded in demonstrating them in three ways, which are different from the one here exhibited; one of which, being founded upon the principle of finding the sum of any powers of the chords and versed sines of arches of the Circle increasing arithmetically; together with many curious inferences deducible from the following demonstrations; I may hereafter take the liberty of communicating to the Academy. My only object, in the present communication, has been to demonstrate the propositions, as they were proposed in the aforesaid work.

THEODORE STRONG.

Hamilton College, Sept. 20th, 1814.

Lemma I.

IF the circumference of any circle be divided into any number of equal parts, and any point be taken in it; and if the several arches intercepted between the assumed point, and the points of division, reckoned

around the circle in the same direction, be multiplied by any whole number, less than the number of the parts into which the circumference is divided; then, the extremities of these multiple arches will divide the circumference into equal parts.

DEMONSTRATION.

Let O (Plate III. Fig. 1.) be any circle, the circumference of which is divided into any number of equal parts, at the points a, b, c, &c.; and let any point, at A, be taken in the circumference, thus divided. Let P denote the circumference, and n, the number of the parts into which it is divided: then $\frac{P}{n}$ = any one of the parts.

Since Aa is such a part of $\frac{P}{n}$, as may be expressed by some fraction, let $\frac{r}{s}$ denote that fraction in its lowest terms: then Aa = $\frac{rP}{sn}$. Let $\frac{rP}{s}$ be denoted by Q: then, the arch Aa = $\frac{Q}{n}$; the arch Aab = $\frac{Q+P}{n}$; the arch Aabc = $\frac{Q+2P}{n}$, &c.; since the arches increase by the addition of $\frac{P}{n}$. Let the arches $\frac{Q}{n}$, $\frac{Q+P}{n}$, $\frac{Q+2P}{n}$, &c. be multiplied by any number m, less than n: then will they be represented by $\frac{mQ}{n}$, $\frac{mQ+mP}{n}$, $\frac{mQ+2mP}{n}$, &c.

Case I. Let the numbers m and n, have no common measure.

Set off from A, towards b, the arch AaB, equal to $\frac{mQ}{n}$; and from B, divide the circumference into as many equal parts as it was divided into at first.

Now, since the arch AaB, or its equal $\frac{mQ}{n}$, belongs to all the arches $\frac{mQ}{n}$, $\frac{mQ+mP}{n}$, $\frac{mQ+2mP}{n}$, &c.; if it be taken away from each of them, the remainders, o, $\frac{mP}{n}$, $\frac{2mP}{n}$, &c. will express the distances at which the aforesaid arches will respectively terminate, from B. Whence the first arch terminates at

B; and they will all terminate in the points at which the circumference was last equally divided; since m, $2m$, &c. are all whole numbers; and $\frac{P}{n}$ = one of the parts, into which the circumference was last divided.

Since m, and n, are prime to each other, and the several numbers 1, 2, 3, &c., until we come to n, are each less than n; it follows that $\frac{m}{n}$, $\frac{2m}{n}$, $\frac{3m}{n}$, &c., until we come to $\frac{nm}{n}$, cannot either of them be a whole number. Therefore, no one of the arches $\frac{mP}{n}$, $\frac{2mP}{n}$, &c., until we come to $\frac{nmP}{n}$, can terminate at the point B. It follows also, that these arches all terminate in different points; for in order that more than one of them should terminate at the same point, some one of them must have terminated at B; when the remaining arches would go on to terminate in the same points, at which those already taken had terminated respectively: (since they increase from B, by the common difference $\frac{mP}{n}$.) Therefore, since the arches o, $\frac{mP}{n}$, $\frac{2mP}{n}$, &c., and $\frac{mQ}{n}$, $\frac{mQ+mP}{n}$, $\frac{mQ+2mP}{n}$, &c., the former series reckoned from B, and the latter from A, do respectively terminate in the same points; and since the arches of the former series do respectively terminate at different points until we come to the arch $\frac{nmP}{n}$; and since they each terminate in some one of the points B, C, D, &c.; and since there are as many arches as points; it follows, that they divide the circumference into as many, and the same parts, as the points B, C, D, &c. divide it. But these points divide it by construction into n equal parts; therefore each series of arches, divides the circumference into n equal parts.

Case II. Let m and n have some common measure, and let t denote their greatest common measure; let $\frac{m}{n}$ be reduced to its lowest terms, by dividing m and n by t; and let $\frac{k}{g}$ denote the fraction thus reduced. Then will the arches $\frac{mQ}{n}$, $\frac{mQ+mP}{n}$, $\frac{mQ+2mP}{n}$, &c., be equal respectively to $\frac{kQ}{g}$, $\frac{kQ+kP}{g}$, $\frac{kQ+2kP}{g}$, &c.

Now, since the fraction $\frac{k}{g}$ is in its lowest terms, k and g are of course prime to each other. Therefore, taking the arch AaB $= \frac{kQ}{g}$, and supposing the circumference to be divided from B, into g equal parts; it may be shewn, as in Case I, that the arches $\frac{kQ}{g}$, $\frac{kQ+kP}{g}$, $\frac{kQ+2kP}{g}$, &c. will divide the circumference at the supposed points, into as many equal parts as there are units in g; when we have taken as many of them as there are units in g: and then the remaining arches will go on to terminate in the same points respectively, as those already taken, until as many are taken, in all, as there are units in $2g$; then the remainder will go on as before, until the number taken is equal to $3g$; and so on, until the number becomes equal to tg, or n. Q. E. D.

Cor. It appears from the demonstration, that when the numbers m and n are prime to each other, the multiple arches divide the circumference into as many equal parts as there are units in n: and that, when m and n are not prime to each other, if the fraction $\frac{m}{n}$, reduced to its lowest terms, be denominated by $\frac{k}{g}$, then the multiple arches divide the circumference into as many equal parts as there are units in g. And if t denote the greatest common measure of the terms of the fraction $\frac{m}{n}$, the number of arches which terminate in each point of division, will be equal to t.

Lemma II.

IF, in the circumference of any circle, there be taken three arches, which have equal differences; it will be, As the diameter, is to the supplementary chord of their common difference; so is the chord of the mean arch, to half the sum of the chords of the extreme arches.

DEMONSTRATION.

Let O (Plate III. Fig. 2.) be any circle; and in its circumference let there be taken any three equidifferent arches, as DK, EH, and FG, so situated that DE = KH, and EF = HG. Draw the chords DK, EH, and FG. From E draw the diameter EN.

Connect the points D and F, by the right line DF; K and G, by KG; and H and N, by HN. From the point L, where the diameter bisects DF, draw LM, parallel to EH or DK, or FG. From M, the point where LM intersects KG, draw MR, parallel to HN. Through F, draw FP, parallel to EN; and through P, the point where FP meets the circumference, draw PQ, parallel to FD.

Since DK and FG are parallel, and DL = LF, and LM is is drawn parallel to DK and FG, it follows that KM = MG; whence $LM = \frac{DK+FG}{2}$. It is manifest also, from the construction, that the arch DEF is the common difference of the assumed arches, and that the line FP, parallel to EN, cuts off the arch FP equal to its supplement, since the angle DFP is a right angle. It is obvious also, that PQ, parallel to FD, meets the diameter EN, in the same point R, in which MR, parallel to HN, meets it. For, from the centre O, draw OT, perpendicular to LM. Since in the triangle LMR, OT is drawn parallel to one of its sides, LT : TM :: LO : OR. But LT = TM; therefore LO = OR; of course, EL = NR. But EL = NQ; wherefore NR = NQ.

Then in the similar triangles LMR, EHN, it will be, EN : (LR or) FP :: EH : (LM or) $\frac{DK+FG}{2}$. Q. E. D.

Corollary I. Hence the chord $FG = \frac{2FP \cdot EH}{EN} - DK = \frac{2FP \cdot EH - DK \cdot EN}{EN}$. In like manner, $DK = \frac{2FP \cdot EH - FG \cdot EN}{EN}$. Therefore, if the chord of the mean of three equidifferent arches, be multiplied by twice the supplementary chord of their common difference; and from their product, there be taken the product of the diameter and the chord of one of the extreme arches; and the difference be divided by the diameter; the quotient will express the chord of the other extreme arch.

Cor. II. Hence the chord of any multiple of an arch may be expressed, in terms of the chord of the arch; its supplementary chord; and the diameter.

Let the points D and K be supposed to coincide; and then let A denote the arch EDH. Then will the arch FDG = 2A. Whence the chord of $2A = \frac{2FP \cdot EH}{EN}$, which is the expression for the chord of the arch FDKG, when the arch DK = *o*.

In like manner, considering A, 2A, 3A, as the arches, the

chord of $3A = \frac{\overline{2FP \cdot EH - EH \cdot EN}^{2}}{EN^{2}}$. In like manner, also, the chord of $4A = \frac{\overline{2FP \cdot EH - 4FP \cdot EH \cdot EN}^{3}}{EN^{3}}$.

In general, the chord of mA, will be equal to $\frac{\overline{2FP \cdot EH}^{m-1} - \frac{m-2}{1} \times \overline{2FP \cdot EH \cdot EN}^{2} + \frac{m-3}{1} \times \frac{m-4}{2} \times \overline{2FP \cdot EH \cdot EN}^{4} - \frac{m-4}{1} \times \frac{m-5}{2} \times \frac{m-6}{3} \times \overline{2FP \cdot EH \cdot EN}^{6} + \&c.}{EN^{m-1}}$; which, if FP be denoted by a; EH, by b; and EN, by d, will become, $\frac{b \times \overline{2a}^{m-1} - \frac{m-2}{1} \times \overline{2a \cdot d^{2}}^{m-3} + \frac{m-3}{1} \times \frac{m-4}{2} \times \overline{2a \cdot d^{4}}^{m-5} - \frac{m-4}{1} \times \frac{m-5}{2} \times \frac{m-6}{3} \times \overline{2a \cdot d^{6}}^{m-7} + \&c.}{d^{m-1}}$; and this series will terminate when the numerator of one of the factors in the co-efficient of the power of $2a$ has become $m - m$.

Lemma III.

IF there be any regular figure circumscribed about a circle; and from any point within the figure, perpendiculars be drawn to all the sides of the figure; the sum of all the perpendiculars will be equal to the multiple of the radius of the circle, by the number of the sides of the figure.

DEMONSTRATION.

Let O (Plate III. Fig. 3) be any circle, and A, B, C, &c. any regular figure circumscribed about it. Let H be any point within the figure, and draw HI, HK, HP, &c. perpendicular to all the sides of the figure; and also draw the right lines HE, HF, HC, HB, &c. to all the angular points of the figure. From O, the centre of the circle, draw OG, ON, &c. to all the points of contact, between the sides of the figure, and the circumference; and also, OE, OD, OA, &c. to all the angular points of the figure.

Now, the area of the triangle $EHF = \frac{HI \cdot EF}{2}$; and the area of $FHC = \frac{HK \cdot CF}{2}$; and so of all the other triangles, whose vertices are at H. But because the figure is regular, the bases

EF, CF, &c. are all equal. Therefore, the sum of the areas of all the triangles EHF, FHC, &c. will be equal to $\frac{EF}{2} \times \overline{HI+HK+\&c.}$ = area of the whole figure.

In like manner, the sum of all the triangles EOD, DOA, &c. whose vertices are at the centre O, is equal to $\frac{DE}{2} \times \overline{OG+ON+\&c.}$; since OG, ON, &c. are respectively perpendicular to the sides ED, DA, &c. But since OG, ON, &c. are all radii of the circle; their sum is equal to the radius, multiplied by the number of the sides of the figure. Therefore, the sum of all the triangles EOD, DOA, &c. $= \frac{DE}{2} \times$ Radius of the circle, $\times$ number of sides of the figure, = Area of the whole figure. But it was before shown, that $\frac{EF}{2} \times$ sum of all the perpendiculars from H = Area of the whole figure. Therefore, (since $\frac{DE}{2} = \frac{EF}{2}$,) the sum of all the perpendiculars from H = Radius of the circle, multiplied into the number of the sides of the figure. Q. E. D.

Note. If n denote the number of the sides of the figure; and R, the radius of the circle; then will the sum of the perpendiculars be equal to nR.

Lemma IV.

IF the circumference of any circle be divided into any number of equal parts; and if from any point in the circumference, right lines be drawn to all the points of division; the sum of the squares of all these lines will be equal to twice the square of the radius of the circle, multiplied by the number of the parts, into which the circumference is divided. Thus, if n denote the number of the parts; and R, the radius of the circle; the sum of the squares of the aforesaid lines, will be equal to $2n\mathrm{R}^2$.

DEMONSTRATION.

Let O (Plate III. Fig. 4) be any circle, the circumference of which is divided into any number of equal parts, at the points

E, D, &c. Take A, any point in the circumference, and draw AE, AD, &c. to all the points of division. Draw the diameter AB; and from all the points E, D, &c. draw EM, DQ, &c. perpendicular to AB. At the points E, D, &c. draw also tangents to the circle; and from A, draw AS, AP, &c. perpendicular to all the tangents.

Since the angles at S and M are both right angles; and the $\angle AES = \angle AEM$; and AE is opposite the angles at S and M; therefore AS = AM. In like manner, AP = AQ. And in general, the perpendicular from A, to any tangent, is equal to the part of AB, intercepted between A, and the point where the perpendicular, let fall from the point of contact of the tangent, intersects AB.

Now $AE^2 = AB \cdot AM$; and $AD^2 = AB \cdot AQ$; and so of all the lines drawn from A to the points of division. Therefore, $AE^2 + AD^2 + \&c. = AB \cdot \overline{AM + AQ + \&c.} = AB \cdot \overline{AS + AP + \&c.}$ But (Lemma 3d.) $AS + AP + \&c. = nR$; since, if the tangents be extended, they will meet, and form a regular figure, circumscribed about the circle, in all cases, except when the circumference is divided into only two equal parts; in which case, the tangents being parallel, the sum of the perpendiculars will be manifestly equal to nR. Therefore, since $AB = 2R$;

$AB \cdot \overline{AS + AP + \&c.} = AE^2 + AD^2 + \&c. = 2nR^2$. Q. E. D.

Cor. Hence, if the arches AE, AED, &c. be multiplied by any number $m \angle n$; the sum of the squares of all the lines drawn from A, to the points where the multiple arches divide the circumference, will be equal to $AE^2 + AD^2 + \&c. = 2nR^2$. For, (Lemma 1st.) the multiple arches, mAE, mAED, &c. divide the circumference into equal parts; the number of which, if m and n are prime to each other, is equal to the number of the parts into which the circumference was at first divided. But if m and n be not prime to each other, let the fraction $\frac{m}{n}$, when reduced to its lowest terms, by dividing m and n by t, its greatest common measure, be denoted by $\frac{k}{g}$. Then (Lemma 1st.) the arches mAE, mAED, &c. divide the circumference into as many equal parts, as there are units in g; and as many arches terminate in each point of division, as there are units in t.

Therefore, when m and n are prime to each other, the sum of the squares of all the lines drawn from A, to the points where the arches mAE, mAED, &c. terminate, will be equal to $2nR^2$.

And when m and n are not prime to each other, the sum of the squares of these lines will be equal to $2g\mathrm{R}^2$. But each one of these lines is to be taken as many times as there are units in t; therefore the sum of the squares of the lines belonging to all the arches, will be equal to $2tg\mathrm{R}^2 = 2n\mathrm{R}^2$.

Proposition I.

LET there be any regular figure inscribed in a circle; and from all the angles of the figure, let there be drawn right lines to any point in the circumference of the circle: the sum of the fourth powers of the chords will be equal to six times the multiple of the fourth power of the radius of the circle, by the number of the sides of the figure.

Thus, if n denote the number of the sides of the figure, and R, the radius of the circle; the sum of the fourth powers of the chords will be equal to $6n\mathrm{R}^4$.

DEMONSTRATION.

Let O (Plate III. Fig. 5) be any circle, and A, B, C, &c. the angular points of any regular figure, incribed within it. Let P be any point in the circumference; and let there be drawn to it, the right lines PC, PB, &c. from all the angular points of the figure.

It is evident, that the angular points of the figure, divide the circumference into equal parts, the number of which is equal to the number of the sides of the figure. Let now, the arches PC, PCB, &c. be multiplied by the number 2. Then (Lemma 1st.) the arches 2PC, 2PCB, &c. will divide the circumference into equal parts. And if right lines be drawn from P, to all the points where the multiple arches terminate; then (Coroll. Lemma 4.) the sum of the squares of the chords, belonging to all the arches, will be equal to $2n\mathrm{R}^2$.

Draw through P, the diameter PQ; and from the points C, B, &c. draw CQ, BQ, &c. Then (Cor. Lemma 2d.) the chord of 2PC will be equal to $\frac{2\mathrm{QC}\cdot\mathrm{PC}}{\mathrm{PQ}}$. But $2\mathrm{QC} = 2\sqrt{\mathrm{PQ}^2 - \mathrm{PC}^2}$; and PQ=2R; and let the chord of the arch 2PC be denoted

by x. Then $x=\frac{2PC\sqrt{4R^2-PC^2}}{2R}$. Hence $x^2=\frac{4R^2\cdot PC^2-PC^4}{R^2}$. And $x^2R^2=4R^2\cdot PC^2-PC^4$; or $PC^4=4R^2\cdot PC^2-x^2R^2$.

In like manner, if y denote the chord of the arch 2PCB, it may be shewn that $PB^4=4R^2\cdot PB^2-y^2R^2$. And so of all the rest. Therefore, PC^4+PB^4+ &c. $=4R^2\cdot PC^2+4R^2\cdot PB^2+$ &c. $-\overline{x^2R^2+y^2R^2+\text{\&c.}}=4R^2\cdot\overline{PC^2+PB^2+\text{\&c.}}-R^2\cdot\overline{x^2+y^2+\text{\&c.}}$ But (Lemma 4th.) PC^2+PB^2+ &c. $=2nR^2$; and (as was shown above) x^2+y^2+ &c. $=2nR^2$. Therefore PC^4+PB^4+ &c. $=4R^2\cdot 2nR^2-R^2\cdot 2nR^2=8nR^4-2nR^4=6nR^4$. Q. E. D.

Cor. Hence, if any regular figure of a greater number of sides than three, be inscribed in a circle; and the same construction remain; the sum of the sixth powers of the chords PC, PB, &c. will be equal to $20nR^6$.

For, let the arches PC, PCB, &c. be multiplied by the number 3; and let the chords of the arches 3PC, 3PCB, &c. be denoted by d, e, &c. respectively. Then (by Cor. 2d. Lemma 2d.) the chord of the arch $3PC=d=\frac{\overline{2QC}^3\cdot PC-PC\cdot PQ^2}{PQ^2}$. Substituting for $\overline{2QC}^2$, and PQ^2, their respective values, $16R^2-4PC^2$, and $4R^2$; it becomes, $d=\frac{16R^2\cdot PC-4PC^3-4R^2\cdot PC}{4R^2}=\frac{12R^2\cdot PC-4PC^3}{4R^2}$. Hence $d^2=\frac{144R^4\cdot PC^2-96R^2\cdot PC^4+16PC^6}{16R^4}$; and $16d^2R^4=144R^4\cdot PC^2-96R^2\cdot PC^4+16PC^6$; or $9R^4\cdot PC^2-6R^2\cdot PC^4+PC^6=d^2R^2$; whence $PC^6=d^2R^4-9R^4\cdot PC^2+6R^2\cdot PC^4$.

In like manner, it may be shewn that $PB^6=e^2R^4-9R^4\cdot PB^2+6R^2\cdot PB^4$. And so of the chords of all the other arches.

Therefore, PC^6+PB^6+ &c. $=R^4\cdot\overline{d^2+e^2+\text{\&c.}}-9R^4\cdot\overline{PC^2+PB^2+\text{\&c.}}+6R^2\cdot\overline{PC^4+PB^4+\text{\&c.}}$ But (Cor. Lemma 4th.) d^2+e^2+ &c. $=2nR^2$; and PC^2+PB^2+ &c. $=2nR^2$: and (by the Proposition) PC^4+PB^4+ &c. $=6nR^4$. Therefore PC^6+PB^6+ &c. $=2nR^6-18nR^6+36nR^6=20nR^6$.

By the same process of reasoning, when the inscribed figure has more than four sides, it may be shown, that the sum of the eighth powers of the chords PC, PB, &c. is equal to $70nR^8$.

In general, let m be any number less than n; then will the sum of the $2m$th powers of the chords PC, PB, &c. be equal to $n \times \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots\cdots 2m-1}{1 \cdot 2 \cdot 3 \cdot 4 \cdots\cdots m} \times 2^m \cdot R^{2m}$; where the numbers 1, 3, 5, 7, &c. are to be continued until the last equals $2m-1$; and the numbers 1, 2, 3, 4, &c., until the last equals m.

Proposition II.

LET there be any regular figure inscribed in a circle; and from all the angles of the figure and the centre of the circle, let there be drawn right lines to any point: the sum of the fourth powers of the lines drawn from the angles of the figure, will be equal to the multiple, by the number of the sides of the figure, of the fourth power of the radius of the circle; together with four times the multiple by the same number, of the fourth power of the line whose square is equal to the rectangle contained by the radius, and the line drawn from the centre; together with the multiple by the same number, of the fourth power of the line drawn from the centre.

*DEMONSTRATION.**

Let O (Plate III. Fig. 6) be any circle, and A, B, C, &c. the angular points of any regular figure inscribed in it. Take any point, as P; and through P, and the centre, draw the diameter POQ, produced if necessary. Draw also PC, PB, &c. from all the angular points; and from C, B, &c. draw CG, BH, &c. perpendicular to the diameter.

When P is taken without the circle, let it be denoted by P'; and when within, by P''.

Now, because the angle CPP' is always obtuse, therefore $CP'^2 = P'P^2 + PC^2 + 2P'P \cdot PG$. And because the angle CPP'' is always acute, therefore $CP''^2 = P''P^2 + PC^2 - 2P''P \cdot PG$. But $PG = \frac{CP^2}{PQ}$. Therefore $CP'^2 = P'P^2 + PC^2 + \frac{2P'P \cdot PC^2}{PQ}$. And

* *Note.* The case where the point falls in the circumference of the circle, was considered in Proposition I., and therefore is here omitted.

$CP'^2 = P'P^2 + PC^2 - \frac{2P'P \cdot PC^2}{PQ}$. By reduction, $CP'^2 = \frac{P'P^2 \cdot PQ + PC^2 \cdot PQ + 2P'P \cdot PC^2}{PQ} = \frac{P'P^2 \cdot 2PO + PC^2 \cdot 2P'O}{2PO} = \frac{P'P^2 \cdot PO + PC^2 \cdot P'O.}{PO}$ And $CP''^2 = \frac{P''P^2 \cdot PO + PC^2 \cdot P''O.}{PO}$ But $P'P = P'O - PO$, and $P''P = PO - P''O$. Therefore, $CP'^2 = \frac{(P'O-PO)^2 \cdot PO + PC^2 \cdot P'O}{PO}$; & $CP''^2 = \frac{(PO-P''O)^2 \cdot PO + PC^2 \cdot P''O}{PO}$.

Wherefore $CP'^4 = \frac{(P'O-PO)^4 \cdot PO^2 + 2(P'O-PO)^2 \cdot PO \cdot PC^2 \cdot P'O + PC^4 \cdot P'O^2}{PO^2}$;

and $CP''^4 = \frac{(PO-P''O)^4 \cdot PO^2 + 2(PO-P''O)^2 \cdot PO \cdot PC^2 \cdot P''O + PC^4 \cdot P''O^2}{PO^2}$.

By the same mode of reasoning, it may be shewn, that $BP'^4 = \frac{(P'O-PO)^4 \cdot PO^2 + 2(P'O-PO)^2 \cdot PO \cdot PB^2 \cdot P'O + PB^4 \cdot P'O^2}{PO^2}$;

and $BP''^4 = \frac{(PO-P''O)^4 \cdot PO^2 + 2(PO-P''O)^2 \cdot PO \cdot PB^2 \cdot P''O + PB^4 \cdot P''O^2}{PO^2}$;

and so of all the other lines drawn from the angular points to P' or P''.

Let n denote the number of the sides of the figure; and R, the radius, or PO. Then $CP'^4 + BP'^4 + \&c. = \frac{R^2 \cdot \overline{(P'O-R)^4 + (P'O-R)^4 + \&c.} + 2(P'O-R)^2 \cdot P'O \cdot R \times \overline{PC^2 + PB^2 + \&c.} + P'O^2 \cdot \overline{PC^4 + PB^4 + \&c.}}{R^2}$ But, $(P'O-R)^4 + (P'O-R)^4 + \&c. = n(P'O-R)^4$. And (Lemma 4th.) $PC^2 + PB^2 + \&c. = 2nR^2$. And (Prop. I.) $PC^4 + PB^4 + \&c. = 6nR^4$. Hence, $CP'^4 + BP'^4 + \&c. = \frac{nR^2(P'O-R)^4 + 4nR^2(P'O-R)^2 \cdot P'O \cdot R + 6nR^4 \cdot P'O^2}{R^2} = n \cdot \overline{(P'O-R)^4 + 4(P'O-R)^2 \cdot P'O \cdot R + 6R^2 \cdot P'O^2} = n \cdot \overline{P'O^4 - 4P'O^3 \cdot R + 6P'O^2 \cdot R^2 - 4P'O \cdot R^3 + R^4 + 4P'O^3 \cdot R - 8P'O^2 \cdot R^2 + 4P'O \cdot R^3 + 6P'O^2 \cdot R^2} = n \cdot P'O^4 + 4P'O^2 \cdot R^2 + R^4 = nR^4 + 4nR^2 \cdot P'O^2 + n \cdot P'O^4$.

In like manner, $CP''^4 + BP''^4 + \&c. = nR^4 + 4nR^2 \cdot P''O^2 + n \cdot P''O^4$. Q. E. D.

Cor. Let m be any number less than n; the sum of the $2m$th powers of the lines will be $nR^{2m} + nx^2A^2R^{2m-2} + nx^4B^2R^{2m-4} + nx^6C^2R^{2m-6} + \&c.$ in which x denotes the line drawn from the assumed point to the centre of the circle, A the co-efficient of the second term of a binomial raised to the mth power, B of the third term, C of the fourth, &c.

Proposition III.

LET there be any regular figure, of a greater number of sides than three, circumscribed about a circle; and from any point in the circumference of the circle, let there be drawn perpendiculars to all the sides of the figure; twice the sum of the cubes of the perpendiculars, will be equal to five times the multiple of the cube of the radius of the circle, by the number of the sides of the figure.

Thus, if n denote the number of the sides of the figure; and R, the radius of the circle; twice the sum of the cubes of the perpendiculars, will be equal to $5nR^3$.

DEMONSTRATION.

Let O (Plate III. Fig. 7) be any circle, and A, B, C, D, &c. the points of contact of the sides of any regular figure, of more than three sides, circumscribed about it. Let P be any point in the circumference; and draw PM, PN, &c. perpendicular to SI, IK, &c. the sides of the figure. Draw the diameter PQ, and also PC, PB, &c. to all the points of contact. And from all the points of contact, draw CG, BH, &c. perpendicular to the diameter PQ.

It may be shewn, (as in Lemma 4th.) that PM = PG; and PN = PH; and so of all the other perpendiculars. But by the nature of the circle, $PM = PG = \frac{PC^2}{PQ} = \frac{PC^2}{2R}$. Hence $PM^3 = \frac{PC^6}{8R^3}$. In like manner, $PN^3 = \frac{PB^6}{8R^3}$. And so of all the other perpendiculars. Hence, by addition, $PM^3 + PN^3 + \&c. = \frac{PC^6 + PB^6 + \&c.}{8R^3}$ But (Cor. Prop. I.) $PC^6 + PB^6 + \&c. =$

$20nR^6$. Hence $PM^3+PN^3+\&c.=\frac{20nR^6}{8R^3}$. And $2\cdot\overline{PM^3+PN^3+\&c.}=\frac{40nR^6}{8R^3}=5nR^3$. Q. E. D.

Proposition IV.

LET there be any regular figure, of a greater number of sides than four, circumscribed about a circle; and from any point in the circumference of the circle, let there be drawn perpendiculars to the sides of the figure; eight times the sum of the fourth powers of the perpendiculars, will be equal to thirty-five times the multiple, by the number of the sides of the figure, of the fourth power of the radius of the circle.

Thus, if n denote the number of the sides of the figure; and R, the radius of the circle; eight times the sum of the fourth powers of the perpendiculars, will be equal to $35nR^4$.

DEMONSTRATION.

Let O (Plate III. Fig. 7) be any circle, and A, B, C, D, E, &c. the points of contact of the sides of any regular figure, of more than four sides, circumscribed about it; and let the same construction be made, as in Proposition III.

By a process of reasoning, similar to that in Prop. III., it may be shewn that $PM^4=\frac{PC^2}{16R^4}$, and $PN^4=\frac{PB^2}{16R^4}$. And so of all the other perpendiculars. Therefore, $PM^4+PN^4+\&c.=\frac{PC^2+PB^2+\&c.}{16R^4}$ But, (Cor. Prop. I.) $PC^2+PB^2+\&c.=70nR^2$. Therefore, $PM^4+PN^4+\&c.=\frac{70nR^2}{16R^4}$. Or, by multiplication, $8\cdot\overline{PM^4+PN^4+\&c.}=\frac{560nR^2}{16R^4}=35nR^4$. Q. E. D.

Cor. In general, let there be any regular figure circumscribed about a circle; let n denote the number of its sides; and let m be any number less than n: let R denote the radius of the circle; and from any point in the circumference, let there be drawn perpendiculars to the sides of the figure: the sum of the mth powers of the perpendiculars will be equal to

$n \times \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots 2m-1}{1 \cdot 2 \cdot 3 \cdot 4 \cdots m} \times R^m$; where the numbers 1, 3, 5, 7, &c. are to be continued until the last be equal to $2m-1$; and the numbers 1, 2, 3, 4, &c., until the last equal m.

For, let the figure in the Proposition be supposed to have n sides. Then, reasoning as before, $PM^m + PN^m +$ &c. $= \frac{PC^{2m} + PB^{2m} +}{2^m R^m}$ &c. But, (Cor. Prop. I.) $PC^{2m} + PB^{2m} +$ &c. $= n \times \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots 2m-1}{1 \cdot 2 \cdot 3 \cdot 4 \cdots m} \times 2^m R^{2m}$. Whence $PM^m + PN^m +$ &c. $= n \times \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots 2m-1}{1 \cdot 2 \cdot 3 \cdot 4 \cdots m} \times 2^m R^{2m} \div 2^m R^m = n \times \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots 2m-1}{1 \cdot 2 \cdot 3 \cdot 4 \cdots m} \times R^m$.

Proposition V.

LET there be any regular figure circumscribed about a circle, of a greater number of sides than three; and from any point within the figure, let there be drawn perpendiculars to the sides of the figure; and likewise let there be drawn a right line to the centre of the circle: twice the sum of the cubes of the perpendiculars will be equal to twice the multiple of the cube of the radius of the circle, by the number of the sides of the figure; together with thrice the multiple, by the same number, of the solid, whose base is the square of the line drawn to the centre, and altitude, the radius of the circle.

Thus, if n denote the number of the sides of the figure; R, the radius of the circle; and d, the line drawn from within the figure, to the centre of the circle; twice the sum of the cubes of the perpendiculars will be equal to $2nR^3 + 3nd^2R$.

*DEMONSTRATION.**

Let O (Plate III. Fig. 8) be any circle, and F, A, B, C, &c. the points of contact of the sides of any regular figure, of more than three sides, circumscribed about it.

Case I. Let V be any point within the circle; and draw VH, VS, &c. perpendicular to KG, GM, &c., the sides of the

* *Note.* The case where the point falls in the circumference of the circle, was considered in Proposition III., and therefore is here omitted.

figure. Through V, draw the diameter PQ. With the centre O, and distance OV, describe a circle. Draw OA, OF, &c. to all the points of contact, from the centre. From V, draw V*h*, V*s*, &c. perpendicular to OA, OF, &c. respectively. From the points *a*, *f*, &c., where the lines OA, OF, &c. cut the circumference of the smaller circle, draw *ap*, *fq*, &c. perpendicular to the diameter PQ.

Since OA is perpendicular to KG, it is parallel to VH; and because V*h* is perpendicular to OA, it is parallel to KG. Therefore, VHA*h* is a parallelogram; and VH=A*h*. And because the perpendiculars V*h*, and *ap*, are drawn from the opposite ends of the same arch V*a*, to the radii, OA and OP, passing through the extremities of the same arch; therefore P*p*=A*h*=VH. In like manner, P*q*=VS; and so of all the other perpendiculars.

Now it is manifest, that the radii OA, OF, &c. divide the circumference of the smaller circle, at the points *a*, *f*, &c. into as many equal parts, as they do that of the greater; and moreover, that the lines V*p*, V*q*, &c. are respectively equal to perpendiculars drawn from V, to the sides of a regular figure circumscribed about the smaller circle, by drawing tangents to it at the points *a*, *f*, &c.; as was shewn in the demonstration of Lemma 4th.

Now $Pp=PV+Vp=PO-VO+Vp=R-d+Vp$. Hence, $Pp^3=(\overline{R-d}+Vp)^3=(R-d)^3+3(R-d)^2\cdot Vp+3(R-d)\cdot Vp^2+Vp^3$. In like manner, $Pq^3=(R-d)^3+3(R-d)^2\cdot Vq+3(R-d)\cdot Vq^2+Vq^3$: and so of all the rest.

Hence, $Pp^3+Pq^3+\&c.=(R-d)^3+(R-d)^3+\&c.+3(R-d)^2\cdot\overline{Vp+Vq+\&c.}+3(R-d)\cdot\overline{Vp^2+Vq^2+\&c.}+Vp^3+Vq^3+\&c.$

But $(R-d)^3+(R-d)^3+\&c.=n(R-d)^3$; and (Lemma 3d.) $Vp+Vq+\&c.=nd$, (where d denotes the radius of the smaller circle;) and (Cor. Prop. IV.) $Vp^2+Vq^2+\&c.=\frac{3}{2}nd^2$, ($m$, in the general expression, being in this case 2;) and (Prop. III.) $Vp^3+Vq^3+\&c.=\frac{5}{2}nd^3$.

Therefore, $Pp^3+Pq^3+\&c.=n(R-d)^3+3nd(R-d)^2+\frac{9}{2}nd^2(R-d)+\frac{5}{2}nd^3=n\cdot\overline{R^3-3R^2d+3Rd^2-d^3+3R^2d-6Rd^2+3d^3+\frac{9}{2}Rd^2-\frac{9}{2}d^3+\frac{5}{2}d^3}=n\cdot\overline{R^3+\frac{3}{2}d^2R}$.

Hence, $2\cdot\overline{Pp^3+Pq^3+\&c.}=2\cdot\overline{VH^3+VS^3+\&c.}=2nR^3+3nd^2R$.

CASE II. (Fig. 9.) Let the point V be taken without the circumference of the circle; and let the same construction be made as before.

It may be shewn, as in Case 1st., that $Pp = VH$; and $Pq = VS$; and so of all the other perpendiculars.

Now $Pp = Vp - VP = Vp - (VO - PO) = Vp - (d - R.)$ Hence $Pp^3 = (Vp - \overline{d - R})^3 = Vp^3 - 3Vp^2 \cdot (d-R) + 3Vp \cdot (d-R)^2 - (d-R)^3$. In like manner, $Pq^3 = Vq^3 - 3Vq^2 \cdot (d-R) + 3Vq \cdot (d-R)^2 - (d-R)^3$. And so of all the rest.

Hence, $Pp^3 + Pq^3 + \&c. = Vp^3 + Vq^3 + \&c. - 3(d-R) \cdot \overline{Vp^2 + Vq^2 + \&c.} + 3(d-R)^2 \cdot \overline{Vp + Vq + \&c.} - \overline{(d-R)^3 + (d-R)^3 + \&c.}$

But, (as in Case 1st.) $Vp^3 + Vq^3 + \&c. = \frac{5}{2} nd^3$; and $Vp^2 + Vq^2 + \&c. = \frac{3}{2} nd^2$; and $Vp + Vq + \&c. = nd$; and $(d-R)^3 + (d-R)^3 + \&c. = n(d-R)^3$.

Therefore, $Pp^3 + Pq^3 + \&c. = \frac{5}{2} nd^3 - \frac{9}{2} nd^2(d-R) + 3nd(d-R)^2 - n(d-R)^3 = n \cdot \overline{\frac{5}{2} d^3 - \frac{9}{2} d^3 + \frac{9}{2} d^2R + 3d^3 - 6d^2R + 3dR^2 - d^3 + 3d^2R - 3dR^2 + R^3} = n \cdot \overline{R^3 + \frac{3}{2} d^2R}$.

Hence, $2 \cdot \overline{Pp^3 + Pq^3 + \&c.} = 2 \cdot \overline{VH^3 + VS^3 + \&c.} = 2nR^3 + 3nd^2R$. Q. E. D.

Proposition VI.

LET there be any regular figure of a greater number of sides than four, circumscribed about a circle; and from *any point*, let there be drawn perpendiculars to the sides of the figure; and likewise a right line to the centre of the circle: eight times the sum of the fourth powers of the perpendiculars, will be equal to eight times the multiple by the number of the sides of the figure, of the fourth power of the radius of the circle; together with twenty-four times the multiple by the same number of the fourth power of the line whose square is equal to the rectangle contained by the radius and the line drawn to the centre; together with three times the multiple of the fourth power of the line drawn to the centre of the circle, by the number of the sides of the figure.

Thus, if n denote the number of the sides of the figure; d, the line drawn from the assumed point to the centre of the circle; and R, the radius of the circle; eight times the sum of the fourth powers of the perpendiculars, will be equal to $8nR^4+24nd^2R^2+3nd^4$.

*DEMONSTRATION.**

Case I. (Fig. 10.) Let the point be taken any where, within the figure; and when without the circle, let it be denoted by V; and when within, by V'. Let the same construction be made, as in Proposition V. (excepting that the figure shall have more than four sides;) and let the intersections of the lines drawn perpendicular to PQ, from all the points f, a, b, c, d, &c., when the point is taken within the circle, be denoted by q', p', &c.

It may be shewn, (as in Case 1st. of Proposition V.) that $V'H=Pp'=R-d+V'p'$; and $V'S=Pq'=R-d+V'q'$. And, (as in Case 2d. of the same Proposition,) that $VH=Pp=Vp-(d-R)$; and $VS=Pq=Vq-(d-R)$; and so of all the rest in either case.

Hence, $Pp'^4=(V'p'+\overline{R-d})^4=V'p'^4+4V'p'^3\cdot(R-d)+6V'p'^2\cdot(R-d)^2+4V'p'\cdot(R-d)^3+(R-d)^4$; and $Pq'^4=V'q'^4+4V'q'^3\cdot(R-d)+6V'q'^2\cdot(R-d)^2+4V'q'\cdot(R-d)^3+(R-d)^4$; and so of the rest.

In like manner, $Pp^4=(Vp-\overline{d-R})^4=Vp^4-4Vp^3\cdot(d-R)+6Vp^2\cdot(d-R)^2-4Vp\cdot(d-R)^3+(d-R)^4$; and $Pq^4=Vq^4-4Vq^3\cdot(d-R)+6Vq^2\cdot(d-R)^2-4Vq\cdot(d-R)^3+(d-R)^4$; and so of the rest.

Hence, $Pp'^4+Pq'^4+\&c.=V'p'^4+V'q'^4+\&c.+4(R-d)\cdot\overline{V'p'^3+V'q'^3+\&c.}+6(R-d)^2\cdot\overline{V'p'^2+V'q'^2+\&c.}+4(R-d)^3\cdot\overline{V'p'+V'q'+\&c.}+(R-d)^4+(R-d)^4+\&c.$

And $Pp^4+Pq^4+\&c.=Vp^4+Vq^4+\&c.-4(d-R)\cdot\overline{Vp^3+Vq^3+\&c.}+6(d-R)^2\cdot\overline{Vp^2+Vq^2+\&c.}-4(d-R)^3\cdot\overline{Vp+Vq+\&c.}+(d-R)^4+(d-R)^4+\&c.$

Now, for the reasons assigned in Case 1st. of Proposition V., (by Prop. IV.) $V'p'^4+V'q'^4+\&c.=\frac{35}{8}nd^4$; and (Prop. III.) $V'p'^3+V'q'^3+\&c.=\frac{5}{2}nd^3$; and (Coroll. Prop. IV.) $V'p'^2+V'q'^2+\&c.=\frac{3}{2}nd^2$; and, (Lemma 3d.) $V'p'+V'q'+\&c.=nd$; and $(R-d)^4+(R-d)^4+\&c.=n(R-d)^4$.

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* *Note.* The case where the point falls in the circumference of the circle, was considered in Proposition IV.; and therefore is here omitted.

Therefore, $Pp'^4+Pq'^4+\&c.=\frac{35}{8}nd^4+10nd^3(R-d)+9nd^2(R-d)^2+4nd(R-d)^3+n(R-d)^4$.

In like manner, $Pp^4+Pq^4+\&c.=\frac{35}{8}nd^4-10nd^3(d-R)+9nd^2(d-R)^2-4nd(d-R)^3+n(d-R)^4$. Or, $Pp'^4+Pq'^4+\&c.$

$=n\cdot\overline{\frac{35}{8}d^4+10d^3R-10d^4+9d^2R^2-18d^3R+9d^4+4dR^3-12d^2R^2+12d^3R-4d^4+R^4-4dR^3+6d^2R^2-4d^3R+d^4}=n\cdot\overline{R^4+3d^2R^2+\frac{3}{8}d^4}$.

And $Pp^4+Pq^4+\&c.=n\cdot\overline{\frac{35}{8}d^4-10d^4+10d^3R+9d^4-18d^3R+9d^2R^2-4d^4+12d^3R-12d^2R^2+4dR^3+d^4-4d^3R+6d^2R^2-4dR^3+R^4}$

$=n\cdot\overline{R^4+3d^2R^2+\frac{3}{8}d^4}$.

Hence, $8\cdot\overline{Pp'^4+Pq'^4+\&c.}=8\cdot\overline{V'H^4+V'S^4+\&c.}=8nR^4+24nd^2R^2+3nd^4$.

And $8\cdot\overline{Pp^4+Pq^4+\&c.}=8\cdot\overline{VH^4+VS^4+\&c.}=8nR^4+24nd^2R^2+3nd^4$.

Case II. (Fig. 11.) Let the point V be taken without the figure; and the same construction be made as before.

It may be shewn as before, that $VH=Pp$, and $VS=Pq$, and so of all the rest.

It is manifest also, that when the sides of the figure produced, fall between the assumed point and the figure, the lines Pp, Pq, &c. will fall between P and V; but, that otherwise, they will fall on the other side of P.

Hence, $VH=Pp=VP\circledcirc Vp=Vp\circledcirc(d-R)$; and $VS=Pq=VP\circledcirc Vq=Vq\circledcirc(d-R)$; and so of all the other perpendiculars.

Then, $Pp^4=(Vp\circledcirc\overline{d-R})^4=Vp^4-4Vp^3\cdot(d-R)+6Vp^2\cdot(d-R)^2-4Vp\cdot(d-R)^3+(d-R)^4$; and $Pq^4=Vq^4-4Vq^3\cdot(d-R)+6Vq^2\cdot(d-R)^2-4Vq\cdot(d-R)^3+(d-R)^4$; and so of all the rest.

Hence, $Pp^4+Pq^4+\&c.=Vp^4+Vq^4+\&c.-4(d-R)\cdot\overline{Vp^3+Vq^3+\&c.}+6(d-R)_2\cdot\overline{Vp^2+Vq^2+\&c.}-4(d-R)^3\cdot\overline{Vp+Vq+\&c.}+(d-R)^4+(d-R)_4+\&c.$

But, as in the first case, Vp^4+Vq^4+ &c. $=\frac{35}{8}nd^4$ (by Prop. IV.) and (Prop. III.) $Vp^3+Vq^3+=\frac{5}{2}nd^3$; and, (Cor. Prop. IV.) Vp^2+Vq^2+ &c. $=\frac{3}{2}nd^2$; and, (Lemma 3d.) $Vp+Vq+$ &c. $=nd$; and, $(d-R^4)+(d-R)^4+$ &c. $=n(d-R)^4$.

Therefore, Pp^4+Pq^4+ &c. $=\frac{35}{8}nd^4-10nd^3(d-R)+9nd^2(d-R)^2-4nd(d-R)^3+n(d-R)^4=n\cdot\frac{35}{8}d^4-10d^4+10d^3R+9d^4-18d^3R+9d^2R^2-4d^4+12d^3R-12d^2R^2+4dR^3+d^4-4d^3R+6d^2R^2-4dR^3+R^4=n.\ R^4+3d^2R^2+\frac{3}{8}d^4$.

Hence, $8\cdot Pp^4+Pq^4+$ &c. $=8\cdot VH^4+VS^4+$ &c. $=8nR^4+24nd^2R^2+3nd^4$. Q. E. D.

Cor. In general, let there be any regular figure circumscribed about a circle; and let n denote the number of its sides: let m be any number less than n; and R the radius of the circle; and from any point, (within the figure, if m be an odd number; but if even, from *any* point, either within or without,) let there be drawn perpendiculars, to the sides of the figure; and likewise a right line to the centre of the circle; and let v denote the line drawn to the centre; and let a be the co-efficient of the third term of a binomial, raised to the mth power; b, the co-efficient of the fifth term; c, the co-efficient of the seventh term, and so on: the sum of the mth powers of the perpendiculars will be equal to $nR^m+nAv^2R^{m-2}+nBv^4R^{m-4}+nCv^6R^{m-6}+$ &c.; substituting A for $a\times\frac{1}{2}$; B, for $b\times\frac{1\cdot3}{2\cdot4}$; C, for $c\frac{1\cdot3\cdot5}{2\cdot4\cdot6}$, and so on. Which expression may be shewn to be true, by reasoning for the 5th, 6th, &c. powers of the perpendiculars, in the same manner as in the two preceding propositions.

Note. The odd powers of the perpendiculars are confined to the figure, because, otherwise, in summing up, (as in Case 2d of this Prop.) part of the odd powers of the lines Vp, Vq, &c. will be affirmative, and part negative; that is, part to be added, and part subtracted; and their sum cannot be found.

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